

Rescattering and finite formation time effects in electro-disintegration of the deuteron in the cumulative region

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Abstract. The role of rescattering due to the final state interaction (FSI) and the influence of the finite formation time (FFT) on the inclusive $D(e, e')X$ and exclusive $D(e, e')n$ electro-disintegration of the deuteron are studied in the cumulative kinematical region $x > 1$ and moderate values of the 4-momentum transfer $Q^2 = 2 \div 10$ (GeV/c)². The spins are averaged out. It is found that in the inclusive process the relative magnitude of rescattering steadily grows with x and that at $x = 1.7$ it has the same order as the plane wave impulse approximation (PWIA) contribution, with the finite formation time effects decreasing the rescattering contribution by $\sim 30\%$. In the exclusive process, with increasing momentum transfer, FFT substantially reduces the effects from FSI, although the latter are still appreciable in the region of momentum transfer investigated.

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1 Introduction

High-energy electro-disintegration of the deuteron is a powerful tool to investigate, first, how and when at larger energies and momentum transfers the description in terms of hadrons (nucleons and mesons) transforms into the one in terms of quarks and gluons and, second, the Colour Transparency (CT) effects predicted by QCD. In terms of relevant Feynman diagrams, one expects that whereas at comparatively low energies virtual nucleons and mesons with standard propagators can be used, at large enough virtualities such a description gradually becomes invalid. To clearly see this phenomenon one has to be able to reach high enough virtualities in the process, which can be achieved by choosing the kinematics forbidden for free (on-mass-shell) nucleons, i.e. the so-called *cumulative* kinematics, which corresponds to values of the Bjorken scaling variable larger than one (note that the importance of studying the x -dependence of CT was first stressed in [1]). In the PWIA the cross-sections are directly related to the behaviour of the deuteron wave function. However such a direct relation is broken by FSI, which naively are expected to be large in the cumulative region; it is here that CT effects are to be taken into account. CT predicts that at high Q^2 FSI become small, so that, in principle, choosing both x and Q^2 large enough one can neglect FSI and have access, via the PWIA, to the high-momentum behaviour of the nuclear wave function. It is of particular im-

portance that for high values of x and Q^2 the rescattering energy may be quite small [2], so that only two-nucleon intermediate states can be considered and the simplest Feynman diagrams with only nucleon lines (although at high virtuality) have to be evaluated. In such a kinematical situation CT effects can originate from the virtual excitation of the ejectile.

In this study we investigate the relative role of FSI and CT effects in the deuteron electro-disintegration, using an approximate picture in which the spins are altogether averaged out. Taking the relativistic spin into account, apart from purely technical difficulties, inevitably introduces a variety of unknown off-shell components of both the electromagnetic production vertex and the rescattering amplitude, which deprives the results of any predictive power. To relate our absolute cross-sections to the experimental data we introduce an effective electromagnetic form factor, chosen to reproduce the PWIA results with full relativistic spins [2]. In our approach CT is introduced on the hadronic level using its equivalence to the final formation time (FFT) effect [3].

2 Basic formulas

Separating the leptonic part, the process we are going to study is

$$\gamma^*(q) + d(2p) \rightarrow N(p_1) + N(p_2), \quad (1)$$

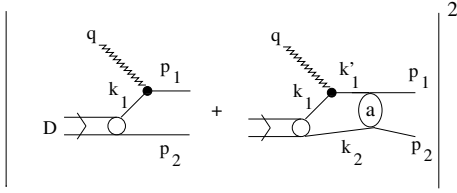


Fig. 1. The square of the sum of the PWIA and rescattering amplitudes proportional to the cross sections

where 4-momenta are denoted in brackets. The total c.m. energy squared in the process is

$$s = (2p + q)^2 = Q^2 \frac{2-x}{x} + M^2, \quad x = \frac{Q^2}{2qp}, \quad (2)$$

where M is the deuteron mass. The cumulative region we are going to study corresponds to $1 < x < 2$. From (2) one concludes that at x close to 2 the c.m. energy remains close to the threshold even if Q^2 is large, so that only the lowest two-nucleon intermediate states can be considered. The calculation of the cross-section for the process (1) then reduces to the evaluation of the square modulus of the two standard diagrams shown in Fig. 1, which correspond to the PWIA and rescattering contributions, respectively.

Our final expression for the inclusive process

$$e(l) + d(2p) \rightarrow e(l') + X \quad (3)$$

can be standardly expressed through the hadronic tensor $W_{\mu\nu}$, which is just the square of the two amplitudes shown in Fig.1 integrated over the momenta of the two final nucleons. It can be shown that in order to obtain the exclusive cross-section for the lepto-disintegration, averaged over the azimuthal angles, it is sufficient to drop this integration.

The PWIA contribution to the amplitude is just the hadronic matrix element of the electromagnetic current

$$J_{\mu}^{PWIA} = 2\gamma(Q^2)K_{\mu}G(k_1^2), \quad (4)$$

where γ is the effective electromagnetic form-factor of the (scalar) nucleon, the vector K is chosen to guarantee conservation of the electromagnetic current, and $k_1 = 2p - p_2 = p_1 - q$ is the momentum of the active nucleon before the interaction. The quantity $G(k_1^2)$ is the relativistic deuteron wave function, which in principle can be sought as a solution of the Bethe-Salpeter equation for the deuteron. However the relativistic potential for this equation is unknown at high momentum transfers and virtualities, so rather than use forms for this potential fitted to comparatively low energy and momentum transfer scattering, we directly approximate G using as a guide its non-relativistic limit. To calculate the rescattering contribution we choose a system with $q_{\perp} = 0$ (“lab” system for reaction (1)) which simplifies the integrations over the intermediate nucleon momenta. In this system the integration over k_{2-} puts either the spectator or the active nucleon on the mass-shell, depending on whether k_{2+} is

lower or higher than $P_+ + q_+$. In both cases, as in the PWIA contribution, the integrand involves only the relativistic deuteron wave function with one of the nucleons on its mass shell.

As a result we find the rescattering contribution to the matrix element of the hadronic current in the form

$$J_{\mu}^{resc} = 2\gamma(Q^2) \int dV(k_1, k_2) K_{\mu} G(v) \frac{a(k_2|p_2)}{m^2 - (k_1 + q)^2}, \quad (5)$$

where $k_1 + k_2 = 2p$, $K = k_1 + q(k_1 q)/Q^2$ and a is the rescattering amplitude, with dV and v having different forms in the two mentioned regions of integrations. All invariant arguments entering the integrand in (5) have to be expressed through the light-cone integration variables, taking into account that either $k_2^2 = m^2$, or $k_1^2 = m^2$.

The total hadronic tensor is obtained as

$$W_{\mu\nu} = \int dV(p_1, p_2) (J_{\mu}^{PWIA} + J_{\mu}^{resc})(J_{\nu}^{PWIA} + J_{\nu}^{resc})^*, \quad (6)$$

where $dV(p_1, p_2)$ is the standard invariant phase volume for the produced nucleons. As far as CT is concerned, we introduce it via the FFT of the hit hadron, which manifests itself through a dependence of the scattering amplitudes and vertexes on the virtuality of the hit hadron after γ^* absorption (see [3]). We assume that the effect of this dependence can be modelled by a monopole form-factor, which is equivalent to changing the ejectile propagator as follows

$$\frac{1}{m^2 - k_1'^2} \rightarrow \frac{1}{m^2 - k_1'^2} - \frac{1}{m^{*2} - k_1'^2}. \quad (7)$$

This substitution is equivalent to assuming that there are two different ejectile states with masses m and m^* whose contribution to rescattering cancels out in the limit of high momenta, in agreement with the underlying ideas of CT [3]. Following [3], [4], we choose for the mass m^* the value 1.8 GeV.

3 Numerical calculations

We parametrize the relativistic deuteron wave function $G(k^2)$ using as a guide the nucleon density in the deuteron at comparatively low momenta. In the non-relativistic limit one obtains

$$G^2(k^2) = 2M(2\pi)^3 |\Psi(\mathbf{k}^2)|, \quad \mathbf{k}^2 = \frac{1}{2}(m^2 - k^2 - M\epsilon), \quad (8)$$

where ϵ is the deuteron binding energy. For $|\Psi|^2$ we have taken the form which corresponds to the AV14 interaction [5]. As for the rescattering amplitude, it was chosen in the form

$$a(s, t) = (\alpha + i)\sigma^{tot}(s)\sqrt{s(s - 4m^2)}e^{bt}, \quad (9)$$

with the values of the parameters σ^{tot} , α and b taken from [6].

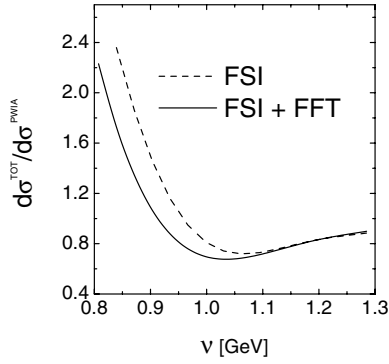


Fig. 2. Ratio of the total inclusive cross section $d\sigma^{TOT} \equiv d\sigma^{TOT}/dE'd\Omega'$ to the PWIA cross section $d\sigma^{PWIA} \equiv d\sigma^{PWIA}/dE'd\Omega'$. In the *dashed curve* $d\sigma^{TOT}$ includes only rescattering effects, whereas in the *full curve* both rescattering and FFT effects are present

As already mentioned, our main goal was to estimate the magnitude of the FSI and the influence of CT (or, equivalently, FFT) in the cumulative region. Accordingly, our basic quantities to be calculated are the ratios of the rescattering to the PWIA contributions. We expect that the error due to neglecting spins is reduced in these ratios. Still, to have a clearer physical picture and to be able to compare with the experimental data, we also tried to calculate absolute values of the cross-sections. To this end we introduced an effective electromagnetic form-factor for the active nucleon, chosen to approximately take into account magnetic interaction of both the proton and neutron. Comparison of our impulse approximation with the results obtained with a full account of spins [2] leads to the choice

$$\gamma^2(Q^2) = \gamma_D^2(Q^2) \frac{2 + \tau(\mu_p^2 + \mu_n^2)}{1 + \tau}, \quad (10)$$

where γ_D is the standard dipole form-factor, $\tau = Q^2/(4m^2)$, and $\mu_{p,n}$ are the anomalous magnetic moments of the proton and neutron. With this choice our PWIA results practically coincide with the ones with full relativistic spins taken into account.

The inclusive cross-sections for the process (3) have been calculated at points corresponding to the experimental data of [7], with the initial electron energy $E = 9.761$ GeV and scattering angle $\theta = 10^\circ$. We have considered values for the final electron energy which cover the region of x in the interval $1.0 < x < 1.71$. Some relevant kinematical values characterizing the chosen points are listed in Table 1.

It can be seen, that at high values of x (high cumulativity) p_{lab} is small, well below the threshold energy for pion production. This justifies our approach based on the assumption that only nucleon degrees of freedom are relevant; however at smaller x p_{lab} grows and such an assumption becomes of disputable validity.

In Fig. 2 the ratios of the full cross section (which includes FSI with or without FFT) to the PWIA cross section is shown. It can be seen that the rescattering contribution, which is very small at $x \sim 1$, steadily grows with

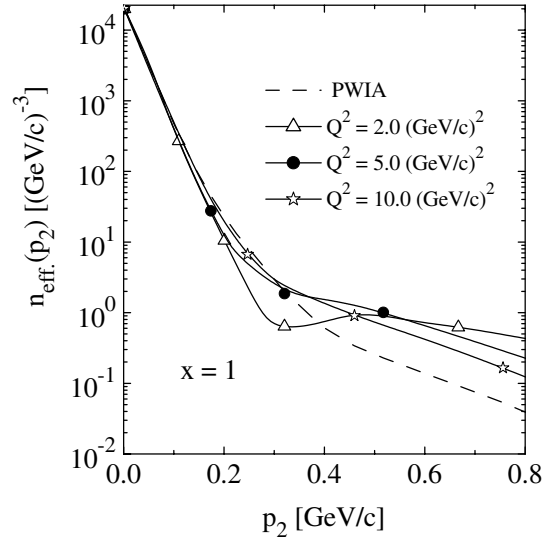


Fig. 3. The effective momentum distribution (11) vs. the neutron recoil momentum $|\mathbf{p}_2| \equiv p_2$ at $x = 1$. The *dot-dashed curve* represents the PWIA result, whereas the *other curves* include also rescattering and FFT effects at various values of Q^2

Table 1. Some kinematical variables in the inclusive electrodisintegration cross section corresponding to the kinematics of of [7], *viz* incident electron energy $E = 9.761$ GeV and scattering angle $\theta = 10^\circ$. ν is the energy transfer, x the Bjorken scaling variable, Q^2 the square 4-momentum transfer, s the produced invariant mass, and p_{lab} the momentum of the struck nucleon in the system where the spectator is at rest p_{lab} . Note that the inelastic threshold corresponds to $s \simeq 4$ GeV² ($p_{lab} \simeq 0.8$ GeV) (cf. [2])

| ν , GeV | x | Q^2 , (GeV/c) ² | s , GeV ² | p_{lab} , GeV/c |
|-------------|------|------------------------------|------------------------|-------------------|
| 0.826 | 1.71 | 2.65 | 3.96 | 0.73 |
| 0.872 | 1.61 | 2.64 | 4.14 | 0.88 |
| 0.930 | 1.50 | 2.62 | 4.38 | 1.06 |
| 0.987 | 1.41 | 2.60 | 4.61 | 1.21 |
| 1.056 | 1.30 | 2.58 | 4.89 | 1.40 |
| 1.137 | 1.20 | 2.56 | 5.22 | 1.61 |
| 1.228 | 1.10 | 2.53 | 5.58 | 1.82 |
| 1.332 | 1.00 | 2.50 | 6.00 | 2.07 |

x , reaching an order of about 50% already at $x \sim 1.3$. The relative role of FFT (or CT) also rises with x . At the maximum value of x studied, $x = 1.71$, FFT effects decrease FSI by $\sim 30\%$. This however does not make FSI smaller than the PWIA contribution, so that they cannot be neglected at all.

Let us now discuss the exclusive cross-section $d(e, e')pn$. In order to minimize the error of neglecting spins, we consider the reduced cross-section

$$n_{eff}(|\mathbf{p}_2|) \equiv n_{eff}(|\mathbf{p}_2|, x, Q^2) = |\Psi(|\mathbf{p}_2|)|^2 \frac{\sigma_{excl}}{\sigma_{excl}^{PWIA}}, \quad (11)$$

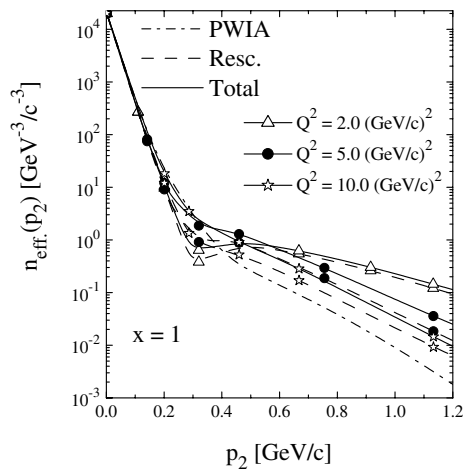


Fig. 4. The effective momentum distribution (11) at $x = 1$ which includes rescattering and FFT effects (*full*), and rescattering effects only (*dashed*); the difference between the *full* and *dashed* curves is due to FFT effects; the *dot-dashed* curve represents the PWIA result

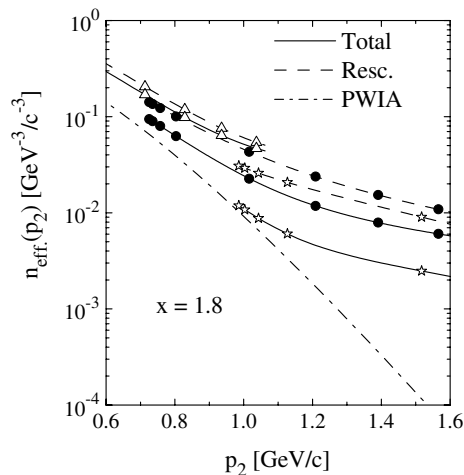


Fig. 5. The same as in Fig. 4 at $x = 1.8$

which in PWIA reduces to the input nucleon momentum distribution in the deuteron. Here p_2 is the momentum of the unobserved nucleon (the missing momentum). Figure 3 shows the effective momentum distributions *vs* the missing momentum at $x = 1$, calculated taking rescattering and FFT into account. Calculations have been performed at fixed values of x and Q^2 and different missing momenta \mathbf{p}_2 . Due to energy conservation, this corresponds to different values of the angle between $|\mathbf{q}|$ and $|\mathbf{p}_2|$.

In Figs. 4 and 5 the effects of rescattering and FFT are shown separately at $x = 1$, and in the deep kinematical region, $x = 1.8$. It can be seen from Fig.4 that at $x = 1$ FFT effects, as expected, increase with Q^2 , in agreement with the results obtained for the process ${}^4\text{He}(e, e'p){}^3\text{H}$ ([8]). At $x = 1.8$ (Fig. 5) one observes that with the growth of the missing momentum, FSI appreciably decrease due to FFT, which makes the distorted cross-section more similar to the PWIA one.

4 Conclusions

Using the Feynman diagram technique we have calculated the rescattering contribution to the cross-section of inclusive and exclusive deuteron electro-disintegration, paying particular attention to the cumulative region $x > 1$. Our main goal has been to study CT effects, which we have introduced via the FFT of the struck nucleon. In our calculations spins has been averaged out and all particles were treated as scalar ones. The error introduced by this approximation is expected to cancel to a large extent in the ratio of the total cross-section (which includes FSI and FFT effects), with and without FSI and FFT effects, to the PWIA cross section. To be able to compare absolute magnitudes of the cross-sections with the experimental data we used an effective electromagnetic vertex for the active nucleon, chosen to describe the PWIA results with spins.

The results of our calculations show that the relative magnitude of the FSI steadily grows with x . At $Q^2 \sim 2$ (GeV/c) 2 and $x \sim 1.7$, the rescattering contribution raises the cross-section nearly by a factor of 2.5, whereas the introduction of FFT decreases back the cross section by a factor of 1.6, which is evidently not enough to disregard FSI altogether. As expected, at large values of x the influence of FFT grows up: it can be seen from Fig. 5 that at $Q^2 = 10$ (GeV/c) 2 and $|\mathbf{p}_2| \sim 1 \div 1.5$ GeV/c, FFT decreases the pure FSI results by a factor of 4.

In conclusion, we have found that the CT or FFT effects are clearly visible in the electro-disintegration of the deuteron in the cumulative region, which may serve as a tool for their experimental study. We have also found that although FFT effects decrease the effects of the FSI rather substantially at $Q^2 = 2 \div 10$ (GeV/c) 2 , this is by far not sufficient to neglect FSI altogether.

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